Practice Exam 1 (Sep. 21, 2015) **Name:**

1. Computers in a shipment of 100 units contain a portable hard drive (PHD), solid-state memory (SSM), or both, according to the following table: Let A denote the event that a computer has a

SSM \ PHD	Yes	No
Yes	27	63
No	3	7

portable hard drive, and let B denote the event that a computer has a solid-state memory. If one computer is selected randomly, compute

(a) $P(A) = \frac{30}{100} = 0.3$.

(b) $P(B) = \frac{90}{100} = 0.9$.

(c) $P(A' \cap B) = \frac{63}{100} = 0.63$.

(d) $P(A \cap B) = \frac{27}{100} = 0.27$.

(e) $P(A|B) = \frac{27}{90} = 0.3$.

(f) $P(B|A) = \frac{27}{30} = 0.9$.

(g) Are A and B independent? Why?
Yes, because $P(A|B) = P(A) = 0.3$.

2. Consider the contamination discussion at the start of this section. The information is summarized here.

Probability of Failure	Level of Contamination	Probability of Level
0.1	Hight	0.2
0.005	Not hight	0.8

Let F denote the event that the product fails, H denote the event that the chip is exposed to high levels of contamination.

(a) $P(F|H) = 0.1$.

(b) $P(F|H') = 0.005$.

(c) $P(H) = 0.2$.

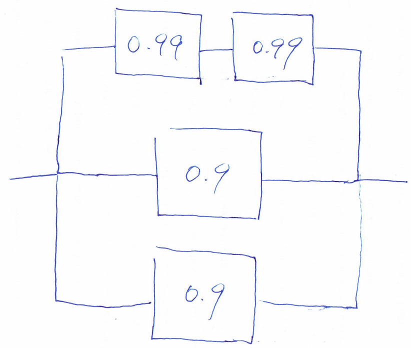
(d) $P(H') = 0.8$.

(e) $P(F \cap H) = P(F|H)P(H) = (0.1)(0.2) = 0.02$.

(f) $P(F \cap H') = P(F|H')P(H') = (0.005)(0.8) = 0.004$.

(g) $P(F) = P(F \cap H) + P(F \cap H') = 0.024$.

3. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown.



Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?

$$1 - [\{1 - (0.99)(0.99)\}\{1 - 0.9\}\{1 - 0.9\}] = 0.999801$$

4. The random variable X has the following probability distribution:

x	2	3	5	8
Probability	0.2	0.4	0.3	0.1

Determine the following:

(a) Find the probability mass function (pmf)

(b) Find the cumulative distribution function (cdf)

(c) $P(X \leq 2) = P(X = 2) = f(2) = 0.2.$

(d) $P(X \geq 3) = P(X = 3) + P(X = 5) + P(X = 8) = f(3) + f(5) + f(8) = 0.4 + 0.3 + 0.1 = 0.8.$

(e) $P(X = 4) = 0.$

(f) $E(X) = \sum_{\text{all } x} xf(x) = 2(0.2) + 3(0.4) + 5(0.3) + 8(0.1) = 3.9.$

(g) $V(X) = E(X^2) - E(X)^2 = \sum_{\text{all } x} x^2 f(x) - E(X)^2$
$= \left\{2^2(0.2) + 3^2(0.4) + 5^2(0.3) + 8^2(0.1)\right\} - (3.9)^2 = 18.3 - (3.9)^2 = 3.09.$

(h) $E(10X + 2) = 10E(X) + 2 = 10(3.9) + 2 = 41.$

(i) $V(10X + 2) = (10)^2 Var(X) = 309.$

$$\text{(a) } f(x) = \begin{cases} 0.2 & \text{if } x = 2, \\ 0.4 & \text{if } x = 3, \\ 0.3 & \text{if } x = 5, \\ 0.1 & \text{if } x = 8, \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{(b) } F(x) = \begin{cases} 0 & \text{if } x < 2, \\ 0.2 & \text{if } 2 \leq x < 3, \\ 0.6 & \text{if } 3 \leq x < 5, \\ 0.9 & \text{if } 5 \leq x < 8, \\ 1 & \text{if } x \geq 8. \end{cases}$$

5. Heart failure is due to either natural occurrences (87%) or foreign objects (13%). Outside factors are related to induced substances or foreign objects. Natural Occurrences are caused by arterial blockage, disease, and infection. Assume that causes of heart failure for the individuals are independent.

(a) What is the probability that the first patient with heart failure who enters the emergency room has the condition due to natural occurrences?

(b) What is the probability that the fourth patient with heart failure who enters the emergency room is he first one due to nature occurrences?

(c) What is the probability that the fourth patient with heart failure who enters the emergency room is he **third** one due to nature occurrences?

(d) Suppose that 5 patients will visit an emergency room with heart failure. What is the probability that three or more individuals with conditions caused by natural occurrences?

Sol.

(a) X = the number of the **first** patients with heart failure who enters the emergency room has the condition due to natural occurrences.
$X \sim Geom(p = 0.87)$.
$P(X = 1) = f_X(1) = (1 - 0.87)^{1-1}(0.87) = 0.87$.

(b) $P(X = 4) = f_X(4) = (1 - 0.87)^{4-1}(0.87) \approx 0.0019$.

(c) Y = the number of **third** patients with heart failure who enters the emergency room has the condition due to natural occurrences.
$Y \sim NB(r = 3, p = 0.87)$.
$P(X = 4) = \binom{4-1}{3-1}(1 - 0.87)^{4-3}(0.87)^3 \approx 0.2568$.

(d) Z = the number of individuals with conditions caused by natural occurrences out of 5 patients.
$Z \sim B(n = 5, p = 0.85)$.
$P(Z \geq 3) = 1 - P(Z < 3) = 1 - P(Z \leq 2) = 1 - F_Z(2) \approx 1 - 0.0266 = 0.9733$.

6. A state runs a lottery in which six numbers are randomly selected from 56 without replacement. A player chooses six numbers before the state's sample is selected.

(a) What is the probability that the six numbers chosen by a player match all six numbers in the state's sample.

(b) What is the probability that five of the six numbers chosen by a player appear in the state's sample.

(c) If a player enters one lottery each week, what is the expected number of weeks until a player matches all six numbers in the state's sample?

Sol.

(a) X = the number of numbers matching six numbers in the state's sample.

$$P(X=6) = \frac{\binom{6}{6}\binom{56-6}{6-6}}{\binom{56}{6}} = 3.0799 \times 10^{-8}.$$

(b)

$$P(X=5) = \frac{\binom{6}{5}\binom{56-6}{6-5}}{\binom{56}{6}} = 9.2397 \times 10^{-6}.$$

(c) Y = the number of weeks until the player matches all six numbers.
$Y \sim Geom(p = 3.0799 \times 10^{-8})$.
$E(Y) = 1/p = 32,468,436$ (weeks). It is about $624,393$ years.

7. The article "An Association Between Fine Particles and Asthma Emergency Department Visits for Children in Seattle" [Environmental Health Perspectives June, 1999, 107(6)] used Poisson models for the number of asthma emergency department (ED) visits per day. For the zip codes studied, the mean ED visits were 1.8 per day. Determine the following:

(a) Probability of more than one visit in a day.

(b) Probability of fewer than one visit in a **week**.

(c) Number of days such that the probability of at least one visit is 0.9.

(d) Instead of a mean of 1.8 per day, determine the mean visits per day such that the probability of **at least one** visits in a day is 0.1.

Sol.

(a) X = the number of visitors in a day.
$X \sim \text{Poisson}(\lambda = 1.8)$.
$P(X > 1) = 1 - P(X = 0) = 1 - 0.4624 = 0.5376$.

(b) Y = the number of visitors in a week.
$Y \sim \text{Poisson}(\lambda = 1.8 * 7)$, i.e., $Y \sim \text{Poisson}(\lambda = 12.6)$.
$P(Y < 1) = P(Y = 0) = 3.3720 \times 10^{-6}$.

(c) Z = the number of visitors in **t** days.
$Z \sim \text{Poisson}(\lambda = 1.8 \times t)$, i.e., $Y \sim \text{Poisson}(\lambda = 1.8t)$.

$$\begin{aligned} & 0.9 = P(Z \geq 1) = 1 - P(Z = 0) = 1 - \frac{[1.8t]^0 e^{-1.8t}}{0!}. \\ \iff \quad & \ln 0.1 = -1.8t. \end{aligned}$$

Then we have $t \approx 1.2792$ days.

(d) W = the number of visitors in a days with $E(W) = \lambda$.
$W \sim \text{Poisson}(\lambda)$.

$$\begin{aligned} & 0.1 = P(W \geq 1) = 1 - P(W = 0) = 1 - \frac{\lambda^0 e^{-\lambda}}{0!} \\ \iff \quad & \ln 0.9 = -\lambda. \end{aligned}$$

Then we have $\lambda \approx 0.1054$ and $E(W) = \lambda = 0.1054$.